Department of Mathematics and Statistics
Indian Institute of Technology Kanpur
MTH101AR Quiz 2 A
April 10, 2013

Roll No: ……………………………………….. **Time: 30 Min**
Marks: 15

Name: ………………………………………………….

1. Use Pappus Theorem to determine the centroid of the part of disc $D=\{(x,y): x^2+y^2 \le 4\}$ lying in the first quadrant. (5)

Solution: Let (x^*, y^*) be the centroid of part S of D lying in the first quadrant. By Pappus Theorem:
Volume generated by revolving S about x-axis = $2\pi \times y^* \times Area\, of\ \ S$

$$\Rightarrow \frac{2\pi\times 8}{3} = 2\pi y^* \times \frac{1}{4}\pi \times 4 \ \Rightarrow y^* = \frac{8}{3\pi}\ .$$ (3 marks)

Since S is symmetric about the line $y=x$, $x^* = y^* = \dfrac{8}{3\pi}$. (2 marks)

$$\Rightarrow Centroid\ \ of\ \ S = (\frac{8}{3\pi}, \frac{8}{3\pi})$$

2. Fot the curve $R(t) = \hat{i} - 2t\hat{j} + \frac{1}{3}(2-t)^3\hat{k}$, determine the unit tangent vector and curvature at $t=1$. (5)

Solution: Unit Tangent Vector $T(t) = \dfrac{R'(t)}{\|R'(t)\|} = \dfrac{-2\hat{j}-(2-t)^2\hat{k}}{\sqrt{4+(2-t)^4}} \Rightarrow T(1) = \dfrac{-2\hat{j}-\hat{k}}{\sqrt{5}}$. (2 marks)

Curvature $\kappa(t) = \dfrac{\|a(t)\times v(t)\|}{\|v(t)\|^3} = \dfrac{\left\|2(2-t)\hat{k}\times(-2\hat{j}-(2-t)^2\hat{k})\right\|}{\left\|-2\hat{j}-(2-t)^2\hat{k}\right\|^3} = \dfrac{|4(2-t)|}{[4+(2-t)^4]^{3/2}} \Rightarrow \kappa(1) = \dfrac{4}{5^{3/2}}$ (3 marks)

3. Using the definition of partial derivatives, compute $f_{yx}(0,0)$ for the function

$f(x,\ y) = xy\dfrac{x^2-y^2}{x^2+y^2}\ \ for\ (x,\ y) \ne (0,\ 0)\ and\ \ f(0,0)=0$. Notation: $f_{yx} \equiv \dfrac{\partial}{\partial x}(\dfrac{\partial f}{\partial y})$. (5)

Solution: $f_y(0,\ 0) = \lim\limits_{\Delta y\to 0} \dfrac{f(0,\Delta y) - f(0,0)}{\Delta y} = 0$ (1 mark)

$f_y = \dfrac{x(x^4-y^4-4x^2y^2)}{\left(x^2+y^2\right)^2}\ \ for\ (x,\ y) \ne (0,0)$ (2 marks)

$\Rightarrow f_{yx}(0,0) = \dfrac{\partial}{\partial x}(\dfrac{\partial f}{\partial y})(0,0) = \lim\limits_{\Delta x\to 0} \dfrac{f_y(\Delta x,0) - f_y(0,0)}{\Delta x} = 1$ (2 marks)

Department of Mathematics and Statistics
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MTH101AR Quiz 2 B
April 10, 2013

Roll No: ……………………………………….. **Time: 30 Min**
Marks: 15

Name: …………………………………………………..

1. Use Pappus Theorem to determine the centroid of the arc of circle $C=\{(x,y): x^2+y^2=9\}$ lying in the first quadrant. (5)

Solution: Let (x^*, y^*) be the centroid of arc of circle L of C lying in the first quadrant. By Pappus Theorem,
Surface Area generated by revolving L about x-axis = $2\pi\times y^*\times Length\ of\ L$

$\Rightarrow 2\pi\times 9 = 2\pi y^*\times\frac{2\pi\times 3}{4} \Rightarrow y^*=\frac{6}{\pi}$. (3 marks)

Since S is symmetric about the line $y=x$, $x^*=y^*=\frac{6}{\pi}$. (2 marks)

$\Rightarrow Centroid\ of\ S=(\frac{6}{\pi},\frac{6}{\pi})$

2. Fot the curve $R(t)=\hat{i}+(1+t)\hat{j}+\frac{1}{3}(1+t)^3\hat{k}$, determine the unit tangent vector and curvature at $t=1$. (5)

Solution: Unit Tangent Vector $T(t)=\frac{R'(t)}{\|R'(t)\|}=\frac{\hat{j}+(1+t)^2\hat{k}}{\sqrt{1+(1+t)^4}}\Rightarrow T(1)=\frac{\hat{j}+4\hat{k}}{\sqrt{17}}$. (2 marks)

Curvature $\kappa(t)=\frac{\|a(t)\times v(t)\|}{\|v(t)\|^3}=\frac{\|2(1+t)\hat{k}\times(\hat{j}+(1+t)^2\hat{k})\|}{\|\hat{j}+(1+t)^2\hat{k}\|^3}=\frac{|2(1+t)|}{[1+(1+t)^4]^{3/2}}\Rightarrow\kappa(1)=\frac{4}{17^{3/2}}$ (3 marks)

3. Using the definition of partial derivatives, compute $f_{xy}(0,0)$ for the function

$f(x,\ y)=xy\frac{x^2-y^2}{x^2+y^2}\ for\ (x,\ y)\neq(0,\ 0)\ and\ f(0,0)=0$. Notation: $f_{xy}\equiv\frac{\partial}{\partial y}(\frac{\partial f}{\partial x})$. (5)

Solution: $f_y(0,\ 0)=\lim_{\Delta y\to 0}\frac{f(0,\Delta y)-f(0,0)}{\Delta y}=0$ (1 mark)

$f_x=\frac{y(x^4-y^4+4x^2y^2)}{\left(x^2+y^2\right)^2}\ for\ (x,y)\neq(0,0)$ (2 marks)

$\Rightarrow f_{xy}(0,\ 0)=\frac{\partial}{\partial y}(\frac{\partial f}{\partial x})(0,\ 0)=\lim_{\Delta y\to 0}\frac{f_x(0,\Delta y)-f_x(0,0)}{\Delta y}=-1$. (2 marks)